Lec 1

course contents

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I Laplace transform

$F(t) \xrightarrow{L} F(s)$ $F(s)$ ناتج تكامل (L) لدالة $F(t)$

$$L[f(t)] = \int_0^\infty F(t)\, e^{-st}\, dt$$

Ex: find laplace transform for $f(t) = K$

$$L(K) = \int_0^\infty e^{-st} K\, dt = K \int_0^\infty e^{-st}\, dt = K \frac{e^{-st}}{-s} \Big|_0^\infty = \frac{K}{s}$$

جدول

$F(t)$	$F(s)$
K	K/s
t^n	$n!/s^{n+1}$
$e^{\pm at}$	$1/s \mp a$
$\sin(wt)$	$\frac{w}{s^2+w^2}$
$\cos(wt)$	$\frac{s}{s^2+w^2}$
$\sinh(wt)$	$\frac{w}{s^2-w^2}$
$\cosh(wt)$	$\frac{s}{s^2-w^2}$

Ex: find $\mathcal{L}(\cos^2(t)) = \mathcal{L}\left(\frac{1}{2} + \frac{1}{2}\cos(2t)\right) = \frac{1}{2s} + \frac{\frac{1}{2}s}{s^2+4}$

* $\mathcal{L}(\sin 3t \cos 2t)$ $\quad \sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$

$= \frac{1}{2}\mathcal{L}(\sin 5t + \sin t)$

$= \frac{1}{2}\left(\frac{5}{25+s^2} + \frac{1}{1+s^2}\right)$

$$f(t) = \begin{cases} 0 & 0 \le t < 4 \\ t^2 & t \ge 4 \end{cases}$$

sol

$$\mathcal{L}(f(t)) = \int_0^4 0 \cdot e^{-st}\,dt + \int_4^\infty t^2 e^{-st}\,dt$$

$$= -\frac{t^2}{s}e^{-st}\Big|_4^\infty - \frac{2t}{s^2}e^{-st}\Big|_4^\infty - \frac{2}{s^3}e^{-st}\Big|_4^\infty$$

u	dv
t^2	e^{-st}
$2t$	$-\frac{1}{s}e^{-st}$
2	$\frac{1}{s^2}e^{-st}$
0	$-\frac{1}{s^3}e^{-st}$

1 shift theorem

* $\mathcal{L}[e^{at}f(t)] = F(s-a)$ ex. $\mathcal{L}(e^{2t}+4) = \frac{4!}{(s-2)^5}$

* $\mathcal{L}[\cos^2 3t\, e^{5t}] = \mathcal{L}\left(\cos^2 3t \cdot \frac{e^{2t}-e^{-2t}}{2} e^{5t}\right) =$

$\sinh 2t$

[1] Diff theorem

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s)$$

[3] Integral theorem

$$\mathcal{L}\left[\frac{F(t)}{t^1}\right] = \int_s^\infty F(s)\, ds$$

Ex:

$$f(t) = 3e^{-4t}(\cos 4t - t\sin 4t)$$

sol.

$3e^{-4t}\cos 4t$ $\quad + \sin(4t)$ $\quad 3e^{-4t}$

$\frac{s}{s^2+16}$ $\quad (-1)^1 \frac{d}{ds}\left(\frac{4}{s^2+16}\right)$ $\quad \frac{8s}{(s^2+16)^2}$

$3\frac{s+4}{(s+4)^2+16}$ $\quad - \frac{-8s}{(s^2+16)^2}$ $\quad = \frac{24(s+4)}{[(s+4)^2+16]^2}$

$$L = 3\frac{(s+4)}{(s+4)^2+16} - \frac{24(s+4)}{[(s+4)^2+16]^2}$$

$$f(t) = \frac{1-e^{-t}}{t}$$

$$F(s) = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s+1}\right) ds = \ln s - \ln s+1 \Big|_s^\infty = \ln \frac{s}{s+1}$$

$$= \left(\lim_{s\to\infty} \ln \frac{s}{s+1}\right) - \ln \frac{s}{s+1} = \ln \lim_{s\to\infty} \frac{s}{s+1} - \ln \frac{s}{s+1}$$

$$= \ln 1 - \ln \frac{s}{s+1}$$

$\lim_{s\to\infty}$ لأن أحد حدود التكامل ∞ تتحول إلى